

MULTIMEDIA



UNIVERSITY

STUDENT ID NO

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MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 2, 2016/2017

ETN2126 – INFORMATION THEORY AND ERROR CODING (TE, MCE)

9 MAC 2017
2:30 P.M.- 4:30 P.M.
(2 Hours)

INSTRUCTION TO STUDENT

1. This examination paper consists of **5 pages** including cover page with **4 questions** only.
2. Attempt **ALL FOUR** questions. All questions carry equal marks and the distribution of the marks for each question is given.
3. Please print all your answers in the Answer Booklet provided.

Question 1

- (a) The symbols of a Discrete Memoryless Source (DMS) have been coded into their corresponding codewords, explain the mutual dependency relationships of these codewords being “prefix code”, “uniquely decodable”, and/or “satisfying the Kraft-McMillan inequality”.

[6 marks]

- (b) The Lempel-Ziv (L-Z) algorithm is used to encode the binary sequence 101110010100110.... Design the codebook for the transmission of this binary sequence.

[5 marks]

- (c) Statistical experiments show that the probabilities of occurrence of the DMS symbols $\{s_0, s_1, s_2, s_3, s_4, s_5\}$ are given as $\{0.24, 0.06, 0.06, 0.40, 0.12, 0.12\}$, respectively.

- (i) Design a Huffman code for this source so that the variance of the codeword length is kept minimal.

[6 marks]

- (ii) Draw the decision tree of the obtained code in part (c)(i) above.

[3 marks]

- (iii) If this DMS has a transmission rate of 100 k symbols/sec and the required information rate is at least 600 kbps, determine the minimum extension order of this source to achieve such an information rate. (Assume that the DMS transmission rate is the same for a block of symbols).

[5 marks]

Continued...

Question 2

- (a) Based on statistical measurements of a given channel with the channel input alphabet $A_X = \{x_0, x_1, x_2\}$ and the channel output alphabet $A_Y = \{y_0, y_1, y_2, y_3\}$, the corresponding channel matrix can be written as follows,

$$P = \begin{bmatrix} p(y_0|x_0) & p(y_1|x_0) & p(y_2|x_0) & p(y_3|x_0) \\ p(y_0|x_1) & p(y_1|x_1) & p(y_2|x_1) & p(y_3|x_1) \\ p(y_0|x_2) & p(y_1|x_2) & p(y_2|x_2) & p(y_3|x_2) \end{bmatrix} = \begin{bmatrix} 0.75 & 0.00 & 0.25 & 0.00 \\ 0.00 & 0.75 & 0.00 & 0.25 \\ 0.00 & 0.00 & 1.00 & 0.00 \end{bmatrix}$$

- (i) Draw the channel diagram which maps A_X to A_Y and label the conditional probabilities of P on the corresponding links.

[4 marks]

- (ii) If the input symbols of A_X are equally probable, calculate the mutual information denoted by $I(A_X; A_Y)$.

[8 marks]

- (iii) Find the channel capacity. Justify your answer.

[2 mark]

- (iv) Determine the conditional probabilities $p(x_0|y_0)$, $p(x_1|y_1)$, and $p(x_2|y_2)$.

[4 marks]

- (b) A Gaussian channel can transfer information at a maximum rate of 2 Mbps. If the channel bandwidth is 0.2 MHz, calculate the achievable signal-to-noise ratio (SNR) in dB.

[4 marks]

- (c) Consider a communication channel whose input and output alphabets are A_X and A_Y , respectively. Fill in the empty boxes of the Venn diagram shown in **Figure Q2** with the following: The entropy of channel input $H(A_X)$, the entropy of channel output $H(A_Y)$, the conditional entropies $H(A_X | A_Y)$ and $H(A_Y | A_X)$, the mutual information $I(A_Y; A_X)$, and the joint entropy $H(A_X, A_Y)$.

[3 marks]

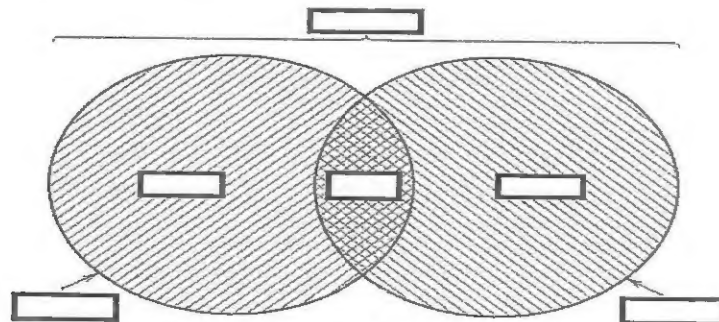


Figure Q2

Continued...

Question 3

(a) Define the following terms,

(i) Hamming weight of a code vector.

[1 mark]

(ii) Hamming distance between two code vectors.

[1 mark]

(iii) Systematic codes.

[1 mark]

(b) The generator matrix \mathbf{G} of a (n, k) systematic linear block code is given below.

$$\mathbf{G} = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

(i) Find the values of n and k for this systematic linear block code.

[2 marks]

(ii) List down all the valid codewords for this code.

[4 marks]

(iii) Find the minimum Hamming distance for this code.

[2 marks]

(iv) How many errors can this code *detect*?

[1 mark]

(v) How many errors can this code *correct*?

[1 mark]

(c) Consider a $(4, 2)$ systematic cyclic code with generator polynomial given as $g(D) = 1 + D^2$.

(i) State and explain the two fundamental properties of a cyclic code.

[4 marks]

(ii) Find the corresponding codewords for the two message vectors $\mathbf{m}_1 = [0 \ 1]$ and $\mathbf{m}_2 = [1 \ 0]$.

[4 marks]

(iii) Calculate the syndrome of the received codeword, $\mathbf{r} = [0 \ 1 \ 1 \ 1]$. Find out whether the received codeword \mathbf{r} contains any errors.

[4 marks]

Continued...

Question 4

- (a) Given that a convolutional encoder has a code rate $r = 1/n$, state the number of modulo-2 adders in the convolutional encoder. Assume that each path has a modulo-2 adder attached to it.

[2 marks]

- (b) Consider the following impulse responses for the paths of a convolutional encoder with constraint length, $K = 3$.

$$g^{(1)} = (1, 1, 1)$$

$$g^{(2)} = (0, 1, 1)$$

$$g^{(3)} = (1, 1, 0)$$

- (i) Find the number of shift registers, modulo-2 adders and paths which are present in the convolutional encoder, assuming that each path has a modulo-2 adder attached to it.

[3 marks]

- (ii) State the rate of this convolutional code.

[2 marks]

- (iii) Give the corresponding generator polynomials of the impulse responses above.

[3 marks]

- (iv) Design the encoder circuit for this convolutional code.

[6 marks]

- (c) The generator polynomials of the rate $\frac{1}{2}$ convolutional encoder in a trellis coded modulation (TCM) system is given as $g^{(1)}(D) = 1 + D^2 + D^3$ and $g^{(2)}(D) = D + D^3$. The most significant bit of the two-bit input message to the TCM system is left uncoded.

- (i) State the 3 basic features of a TCM system.

[3 marks]

- (ii) State a suitable M -PSK modulation scheme for the signal mapper of the TCM system described above.

[1 mark]

- (iii) Sketch the block diagram of the TCM system described above. Include both the convolutional encoder and signal mapper in the block diagram.

[5 marks]

End of Paper